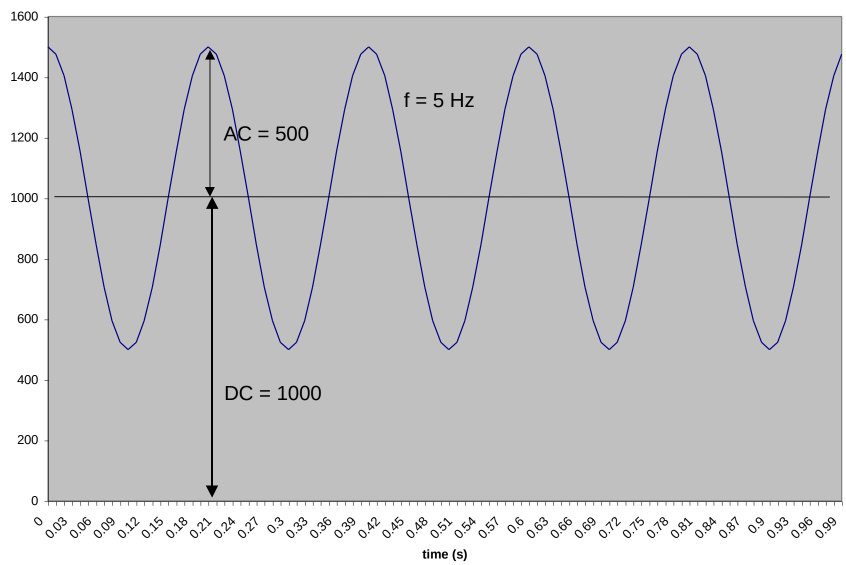
Theoretical

Data collection involves a light source that is set at a certain intensity value. This intensity value is called the direct current, or "DC," component (hearkening back to electrical signal jargon) because it is time-invariant (i.e., "direct"). This source intensity is then amplitude (i.e., intensity) modulated by a cosine wave which oscillates at a given frequency and which has a given phase. The amplitude of this modulation is called the alternating current, or "AC," component (again, back to electrical jargon) because it varies with time. Mathematically, this is expressed like so:

$$\text{Source Signal} = DC_s + AC_s \cos(2\pi f_s t + \phi_s)$$

DC_s = DC signal of the source
AC_s = AC signal of the source modulation
f_s = frequency of the source modulation
ϕ_s = phase of the source modulation

Graphically, it looks like this:



When the light source passes through the head, the light is absorbed and scattered, reducing the DC and AC terms and changing the phase. This can be described like so, assuming that the DC term is affected only by absorbance, not by scattering.

$$L_{HEAD} = DC_L + AC_s \cos(\delta) \cos(2\pi f_s t + \delta)$$
$$DC_L = DC_s \exp(-\varepsilon cl) \text{ [This is Beer's Law]}$$

L_{HEAD} = light through head
DC_s = DC of source
ε = molar absorptivity in L mol^{-1} cm^{-1} for absorbing group (e.g., heme moiety in hemoglobin)
c = concentration of absorbing group
l = pathlength of light
AC_s = AC of source
δ = phase shift of L_{HEAD} relative to the phase of source = $\phi_s - \phi_{\text{light through head}}$
f_s = modulation frequency of source

The detector used to sample the light through the head is **gain modulated** at a frequency, f_d, which is close to the modulation frequency of the source, f_s. This is expressed like so:

$$\text{Gain} = DC_d + AC_d \cos(2\pi f_d t + \phi_d)$$

DC_d = DC of detector
AC_d = AC of detector
f_d = frequency of detector modulation
ϕ_d = phase of detector modulation; this is locked to the ϕ_s, such that $\phi_d = \phi_s$

When the light through the head reaches the detector, the modulated light signal mixes with the modulated gain signal. This is expressed mathematically by multiplying the two signals, resulting in the following:

$$\begin{aligned}\text{Gain x } L_{HEAD} = \;& DC_L\, DC_d\, + \\ & DC_d\, AC_s \cos(\delta) \cos(2\pi f_s t + \delta) + \\ & DC_L\, AC_d \cos(2\pi f_d t + \phi_d) + \\ & AC_s\, AC_g \cos(\delta)\cos(2\pi f_d t + \phi_d)\cos(2\pi f_s t + \delta)\end{aligned}$$

DC_L = DC of light through head
DC_d = DC of detector
AC_d = AC of detector
AC_s = AC of source
δ = phase shift of light through head relative to source
ϕ_d = phase of detector modulation (equals ϕ_s)

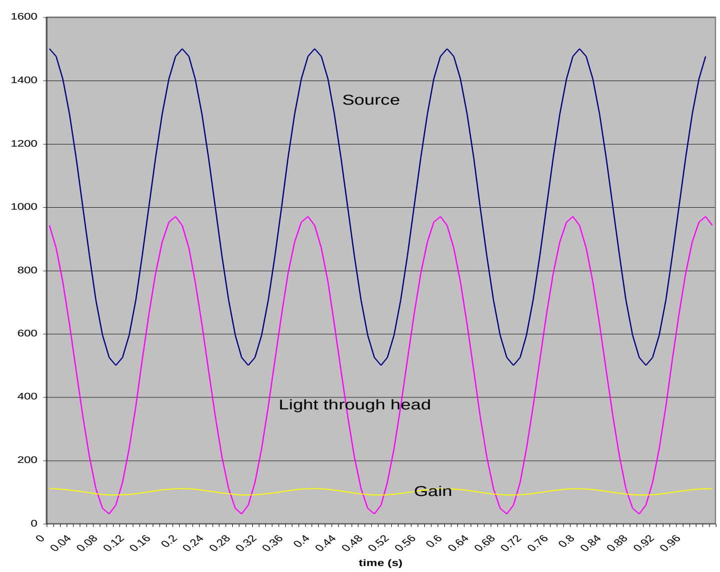
We use a trigonometric function product formula (i.e., cosAcosB = .5cos(A-B) + .5cos(A+B)), applying it to the fourth term, like so:

$$
\begin{aligned}
\text{Gain x } L_{HEAD} = \quad & DC_L\, DC_d + \\
& DC_L\; AC_d \cos(2\pi f_d t + \phi_d) + \\
& DC_d\, AC_s \cos(\delta) \cos(2\pi f_s t + \delta) + \\
& \mathbf{AC_s\, AC_g \cos(\delta)\, .5\cos(2\pi(f_d\text{-}f_s)t + (\phi_d\text{-}\delta))} + \cos(2\pi(f_d+f_s)t+\phi_d+\delta))
\end{aligned}
$$

The first term has all of the information regarding the DC quantities and the term in **bold** has all of the information regarding AC and phase. Moreover, the bolded term oscillates at a much slower frequency than all the other terms. So, we can lowpass filter the signal and sample it at a much slower rate, even as slow as 2.5Δf, where $\Delta f = f_d - f_s$. This is the **cross correlation frequency**.

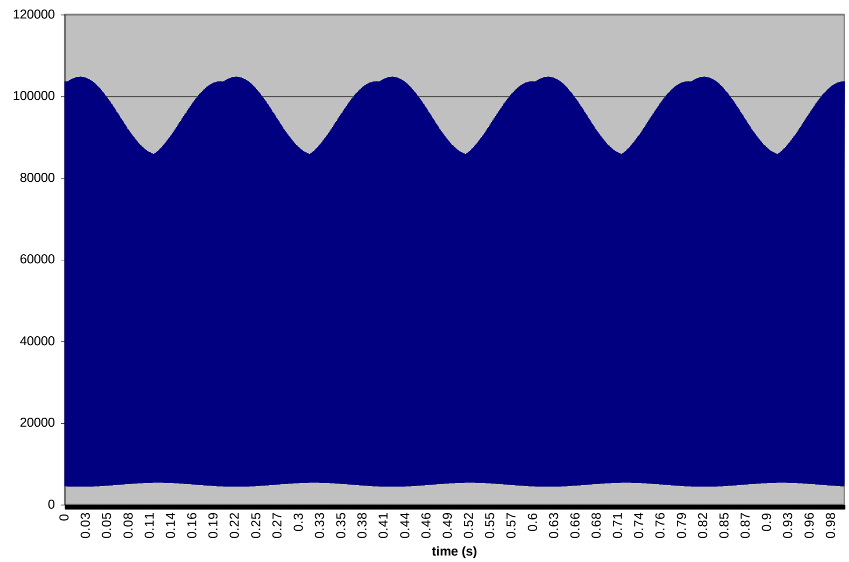
Below, the source, light through head, and detector (gain) are represented graphically, having the following values:

$DC_s = 1000$, $\phi_s = \phi_d = 0$, $\delta = 20$, $f_s = 5$ Hz, $f_d = 5.025$ Hz, $AC_s = 500$
$DC_d = 100$, $AC_d = 10$

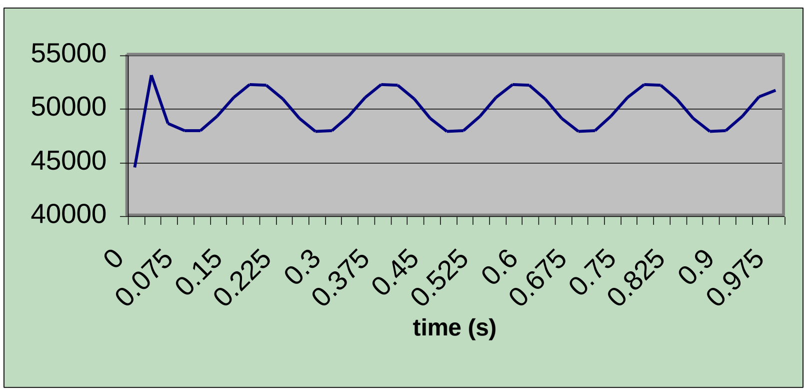


The following graph results from the mixing of a signal and gain with the parameters listed below. Notice the slowly oscillating (5 Hz) signal, seen by the "humps" in the graph. This oscillation occurs at the **cross correlation frequency**. The entire signal can be lowpass filtered and analyzed at this frequency to determine the parameters. Remember that the results need to be corrected by a reference signal, which is simply the source mixed with the gain directly.

$DC_L = 500$, $AC_s = 500$, $\phi_s = \phi_d = 0$, $\delta = 20$, $f_s = 1000$ Hz, $f_d = 1005$ Hz
$DC_d = 100$, $AC_d = 10$



This is the sampled signal, lowpass filtered at 16 Hz and sampled at 40 Hz:



This sampled signal is then analyzed in the following way. Each period is Fourier transformed, and the values at each point in the transforms are averaged. Typically, there will be at least 25 periods collected. The peak at the 0'th ("zeroth") frequency (the first point in the Fourier transform) contains the DC information. The peak at the 5 Hz frequency contains the AC information (i.e., amplitude of the complex number = AC = sqrt((real(number))^2 + (imag(number))^2)), and the phase information (i.e., phase of the complex number = arctan(imag(number)/real(number))). The terms **imag** and **real** refer to taking the imaginary and real parts of the complex number, respectively.